

Vertical social distancing policy (*'Isolamento Vertical'*) is ineffective to contain the coronavirus COVID-19 pandemic

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Abstract

We show, through numerical simulations, that the so-called Vertical Social Distancing health policy ('Isolamento Vertical') is ineffective to contain the COVID-19 pandemic. We present the SEIR-Net model, for a network of social group interactions, as a development of the classic mathematical model of SEIR epidemics (Susceptible - Exposed - Infected (symptomatic and asymptomatic) - Removed). In the SEIR-Net model, we can simulate social contacts between groups divided by age groups and analyze different strategies of social distancing. In the vertical distancing policy, only the elderly are distanced, against the horizontal distancing policy, where all age groups adhere to social distancing. These two scenarios are compared to a control scenario in which no intervention is made to distance people. The vertical distancing scenario is almost as bad as the scenario where no distancing is done at all, both in terms of the number of infected and in the acceleration of the number of cases. On the other hand, horizontal distancing, if applied with the same intensity in all age groups, significantly reduces the total number of infected and "flattening the disease growth curve." Our analysis is done for the municipality of Belo Horizonte, but similar conclusions apply to other cities as well. An R language program is provided.

Resumo

Mostramos, através de simulações numéricas, que o chamado Isolamento Social Vertical é ineficaz para conter a pandemia COVID-19. Apresentamos o modelo SEIR-Net, para uma rede de interações de grupos sociais, como um desenvolvimento do modelo matemático clássico de epidemias SEIR (Suscetível - Exposto - Infectado (sintomático e assintomático) - Recuperado). No modelo SEIR-Net podemos simular contatos sociais entre grupos divididos por faixas etárias e fazer análise de diferentes estratégias de isolamento (ou mais corretamente, distanciamento) social. No distanciamento vertical apenas as pessoas idosas são distanciadas socialmente, e no horizontal pessoas de todas as faixas etárias aderem ao distanciamento. Esses dois cenários são comparados ainda a outro cenário de controle em que nenhuma intervenção é feita para se distanciar socialmente as pessoas. Concluimos que o cenário de distanciamento vertical é quase tão ruim quanto o cenário em que nenhum distanciamento é feito, tanto em termos do número de infectados como na aceleração do número de casos. Por outro lado o distanciamento horizontal, se aplicado com a mesma intensidade em todos os grupos etários, reduz significativamente o número total de infectados e a aceleração do número de casos, "achatando a curva" de crescimento da doença. Nossa análise foi feita para o município de Belo Horizonte, mas conclusões similares valem também para outras cidades. Um programa em linguagem R é disponibilizado.

Introduction

In Brazil, there is a widespread belief that the so-called *Isolamento Vertical*, or vertical social distancing health policy, just restricting social contact with the elderly (and higher risk individuals), would be enough to contain the propagation of the SARS-CoV-2 coronavirus disease (COVID-19). This notion is based on the premise that people under the age of 60 would suffer only mild symptoms and could leave their houses to work and study during the epidemic. However, we have observed an elevated number of hospitalizations, with severe cases and deaths also of people under the age of 60 years old and without underlying diseases. Besides, social distancing is not as a rule 100 percent strict, and the elderly tend to make social contacts during the period, thus increasing the likelihood of being infected. Through this work, we shall use the terms 'social distance' and 'social isolation' interchangeably to indicate a reduction in the intensity of social contact.

Measures of social distance in the COVID-19 pandemic are already proving its effectiveness, favoring the reduction of the number of infected people ([Jefferson, 2009], [Bakker, 2020], [Prem, 2020], [Ganem, 2020], [Sanche, 2020], [Adam, 2020] and [Gallotti, 2020]). Why is it important? Mainly because we want the maximum peak of the epidemic to be minimized, meaning that hospitals are not overcrowded with a large number of people with severe manifestations of COVID-19 requiring intensive care simultaneously. This goal of public health services is popularly known as "flattening the curve" of cases and hospitalizations. If there are not enough hospital beds to serve everyone, many people may die simply from lack of care. Postponing the peak of cases would be potentially beneficial so that health managers could be better prepared, and researchers would find more effective treatments. Therefore, if social distancing can reduce the peak of cases of infected people, and at the same time postponing its occurrence, many lives can be saved.

We will analyze these problems with a mathematical technique for simulating the evolution of epidemics, the SEIR-Net model, obtained by us from a modification of the traditional SEIR model (Susceptible - Exposed - Infected- Removed). In the SEIR model, people *susceptible* to infection randomly come into contact with the SARS-CoV-2 virus becoming *exposed*. After the incubation period, they become *infected* and become able to pass this virus at random to other susceptible people. Infected people can be asymptomatic (have few or no symptoms) or symptomatic (develop typical symptoms of COVID-19 infection). Infected people become, over time, *removed* (a technical term to say that they cannot infect other people and may survive or die). In this model, we use an estimate of the number of unreported cases, based on the number of reported (confirmed) cases. This simplification is due because, in practice, it is not possible to test the entire population at all times. In Brazil, it was estimated that there are at least 20 times more unreported than reported cases. Still, this number now is probably much higher, mainly due to the scarcity of available test kits (see [Takahashi, 2020]). It is important to note that our model assumes that the contacts among persons follow a uniformly random pattern of interaction, and apart from the age groups, there is no spatial (geographic) restriction to social contact – that is embedded into the B transmission parameter, which is determined empirically from the observed data at the beginning of the epidemic.

Some parameters are of interest in this simulation, such as the average incubation time Z , the average infectious period D (for how many days the infected individual is able to infect others), and the fraction of asymptomatic infected individuals which are still capable of infecting others (albeit with less intensity). An important parameter, which does not depend only on the virus, is the transmission rate B ; it depends on the country's health system and the environment in which people live. If the value of B is high, it means that the virus tends to spread more quickly in the population. All COVID-19 parameters used in this work were obtained from the article [Li, 2020] and adapted to the observed case data from Belo Horizonte ([Takahashi, 2020]). Publicly available population data was used ([DATASUS, 2020]).

In the next section, we will build our new Model SEIR-Net with social distancing and network interaction. In the following section, we will present several scenarios simulating different conditions of vertical and horizontal social distancing in Belo Horizonte and study their impact on reducing the number of simultaneously infected people. An R-language program is presented.

The SEIR-Net model

The model proposed in this work is a development of the model used in [Takahashi, 2020], and generalizes the SEIR model proposed in [Duczmal, 2020]. The recent model by [Prem, 2020] also uses the SEIR model with a partitioning of the population into groups and considers their interaction. For a detailed description of these models, see the links in the References section.

The SEIR-Net model divides the population between n social distancing groups and uses the F Contact Fraction Matrix, given by

$$F = \begin{pmatrix} F_{11} & \cdots & F_{1n} \\ \vdots & \ddots & \vdots \\ F_{n1} & \cdots & F_{nn} \end{pmatrix}$$

where the entry F_{ij} indicates the contact intensity of virus transmission from an individual in the group i to an individual in the group j , where $0 \leq F_{ij} \leq 1$. If $F_{ij} = 1$, then the contact is not restricted, and if $F_{ij} = 0$ no individual in the group i can transmit the virus to any individual in the group j . This system forms a connection network between the n groups. In our work, we will use groups formed by age groups in the city of Belo Horizonte. In future work, we will extend this idea to groups divided by income level, place of residence or work, occupation, etc.

How will the COVID-19 epidemic evolve in this case? In the next section we will analyze scenarios with different structures for the F matrix.

Case Studies

In the SEIR-Net model, we can simulate social contacts between groups divided by age groups and analyze different strategies of social distancing.

The population of the municipality of Belo Horizonte, MG, with approximately 2.5 million inhabitants, has the following age distribution interpolated for 2015 ([DATASUS2020]):

0-9 years old: 11.7 %

10-24 years old: 21.8 %

25-59 years old: 52.3 %

60+ years old: 14.2 %

Initially, we present a control scenario in which no distancing intervention is performed. In this case, all elements F_{ij} of the F matrix are equal to 1.

In vertical distancing, only people aged 60+ years old are socially distanced. The F matrix is given by

$$F = \begin{bmatrix} 1 & 1 & 1 & c \\ 1 & 1 & 1 & c \\ 1 & 1 & 1 & c \\ c & c & c & c \end{bmatrix}$$

where $c = 1/k$ means that contacts between individuals aged 60+ years old have a k -fold reduced social contact with individuals of all age groups and vice versa.

Finally, in the scenario of horizontal distancing, individuals of all age groups adhere to distancing. The F matrix becomes

$$F = \begin{bmatrix} c & c & c & c \\ c & c & c & c \\ c & c & c & c \\ c & c & c & c \end{bmatrix}$$

where the value of c depends on the social contact reduction factor. A value of $c = 1/15$ corresponds to the estimated social contact reduction factor for New York City during the end of March, with a 15-fold social contact reduction (see [Bakker2020]). When $c=1$, we go back to the control scenario. As we will see, a value of $c= 0.55$ for the above matrix is consistent with the observed data in Belo Horizonte city.

Below we present the results of the simulations using the SEIR-Net model.

The colored curves (blue (0-9 years old), green (10-24 years old), orange (25-59 years old), and red (60+ years old)) measure the total cumulative values of people who have been infected over time to each age group. The colored dashed horizontal lines indicate the number of persons for each age group, thus showing a maximum ceiling for the number of infected persons in each group.

The numbers in parentheses indicate the approximate percentage of each age group within the population as a whole.

In the control scenario (without distancing) of Figure 1, a maximum of almost 500 thousand simultaneously infected people is reached about 55 days after the beginning of the epidemic. The number of infected people is very high for all age groups. In particular, within the age group of 60+ years old, we would have more than 350 thousand infected people accumulated over the period.

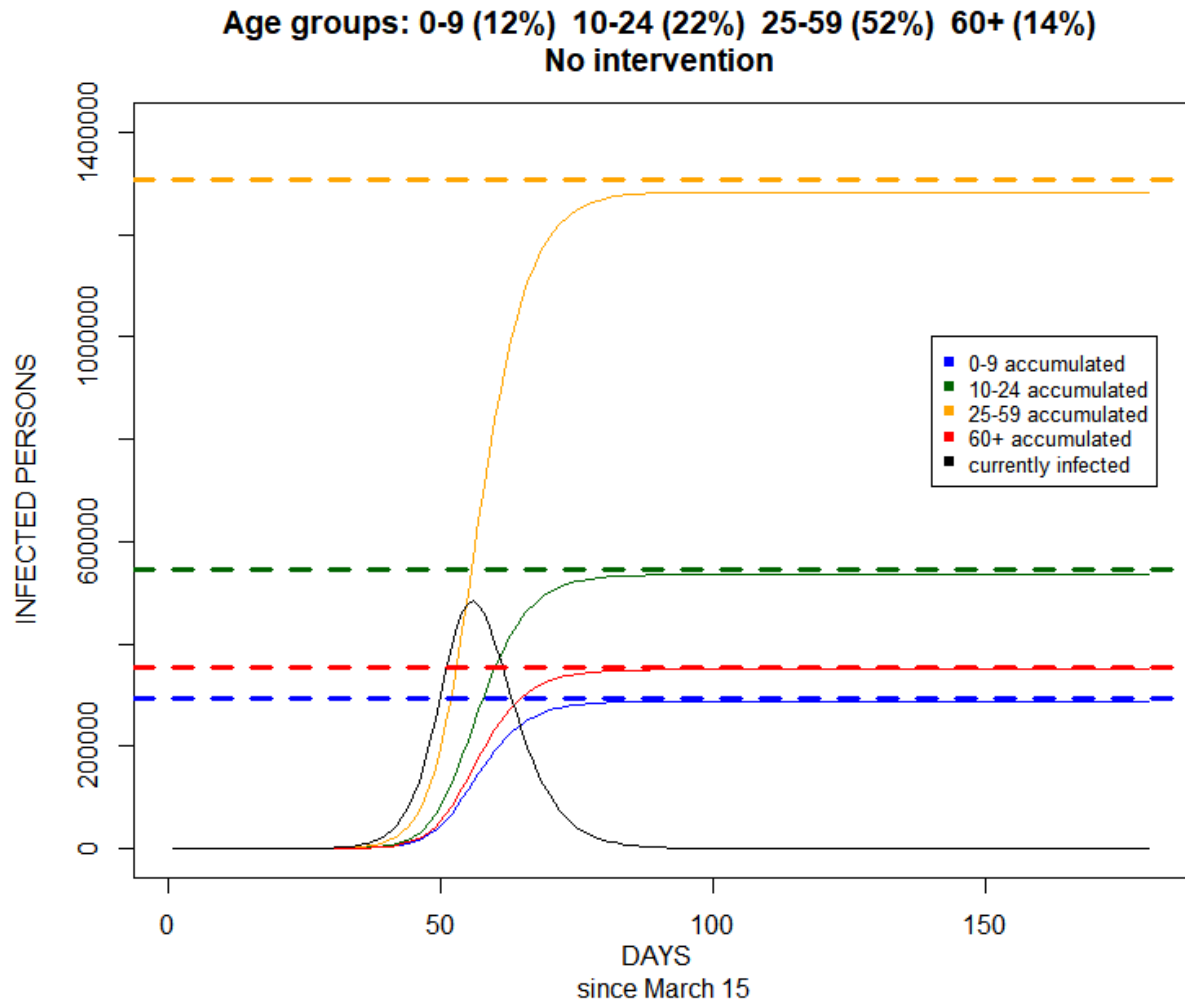


Figure 1: Control scenario without any social distancing, presented for comparison purposes.

In the scenario of vertical distancing, with a 4-fold reduction of social contact intensity factor for only the 60+ years old age group (Figure 2), a maximum of about 400 thousand simultaneously infected people is reached approximately 65 days after the beginning of the epidemic. The number of infected people is very high for all age groups. In the age group of 60+ years old, we would have more than 200 thousand infected people accumulated over the period. In other age groups, virtually everyone has been infected.

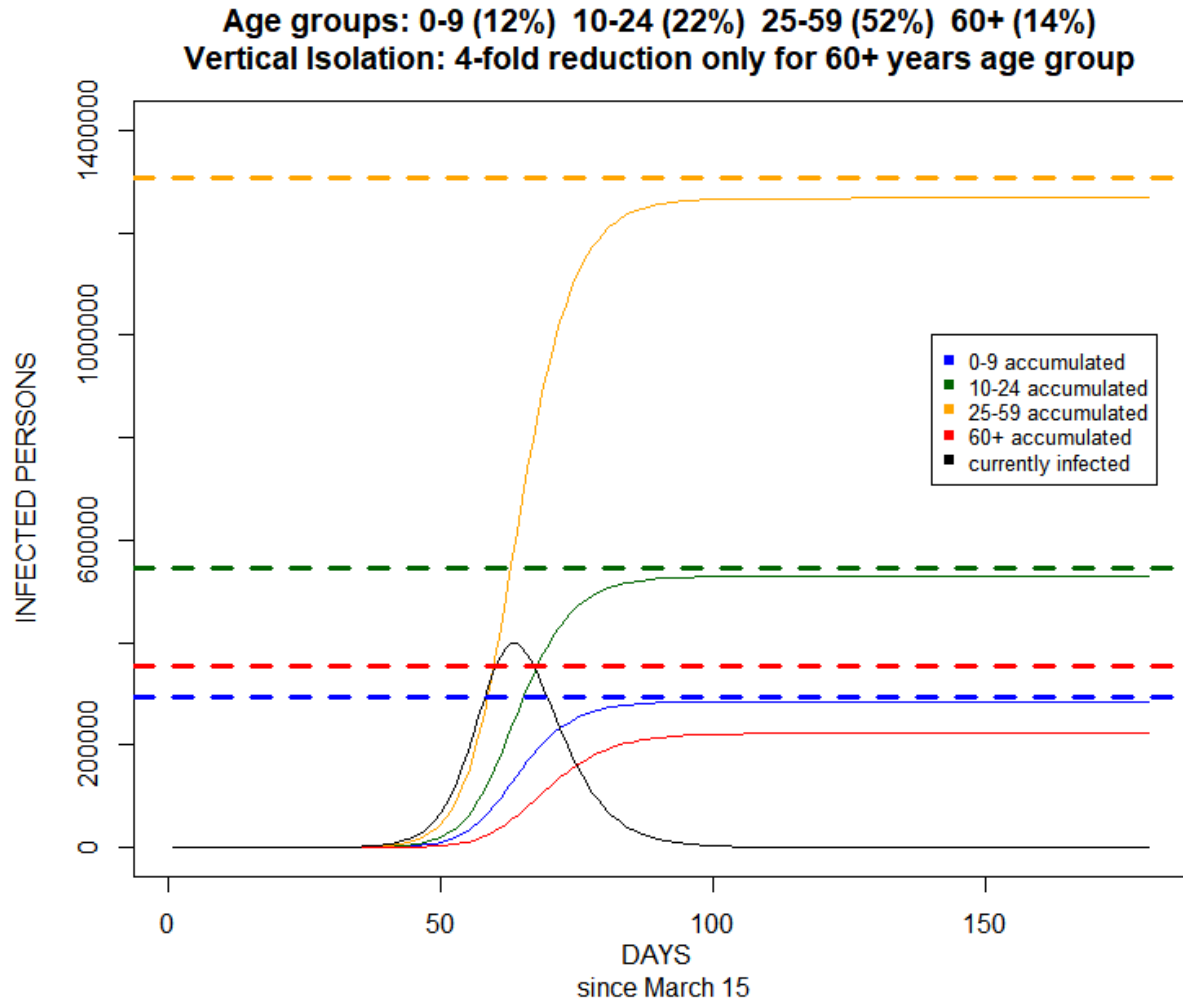


Figure 2: Vertical distancing, only with the 60+ years old age group socially distanced (4-fold). This scenario is almost as unfavorable as the scenario in which there is no distancing at all.

The scenario of horizontal distancing with the same 4-fold social contact intensity factor, for all age groups, is shown in Figure 3. The epidemic does not reach significant dimensions in the first 180 days of simulation. As can be seen in Figure 4, the number of simultaneously infected people only becomes significant about 18 months later, with a relatively small number of simultaneously infected people (less than 10 thousand).

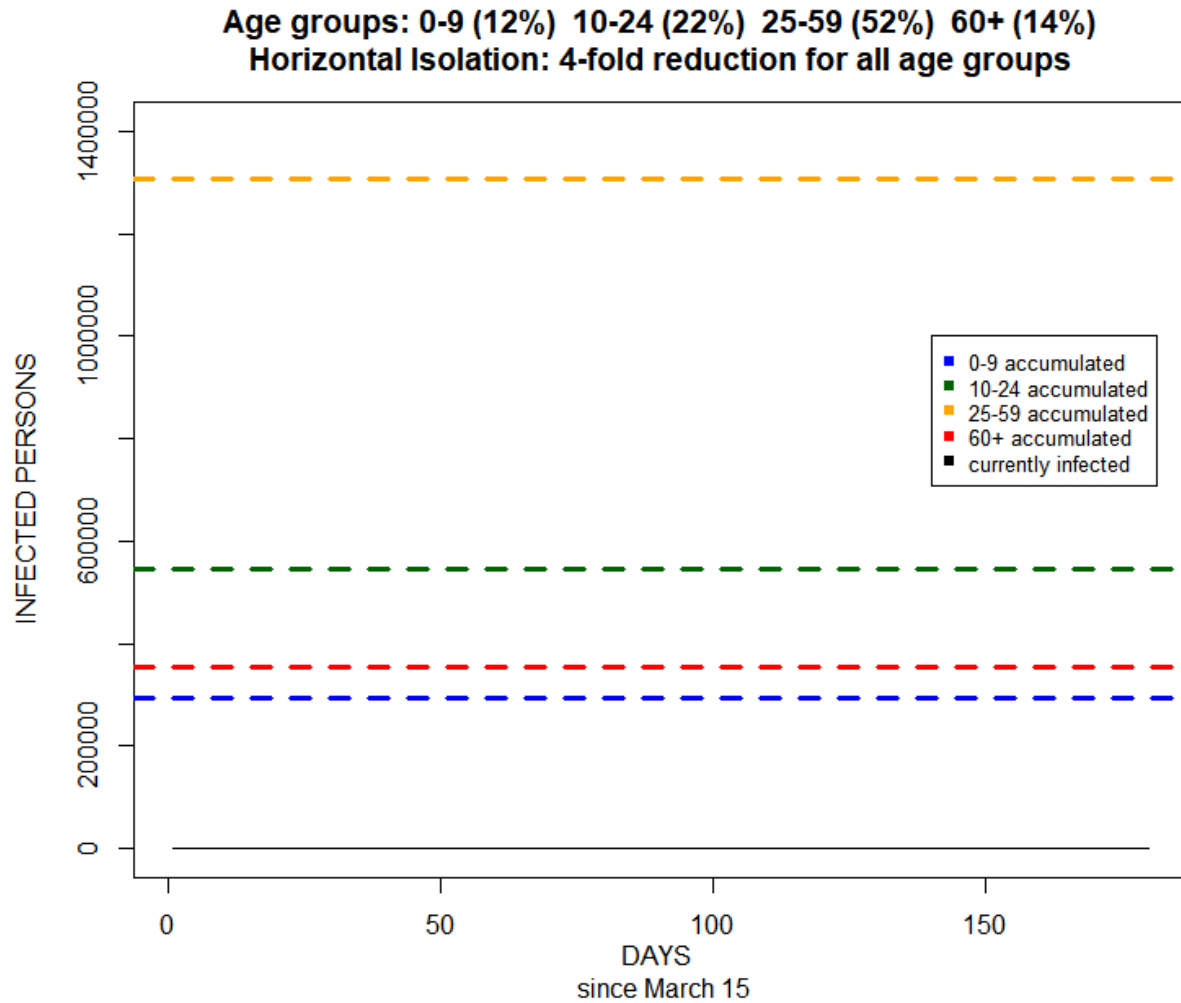


Figure 3: Horizontal distancing, with a 4-fold social contact intensity factor for all age groups. The epidemic does not reach significant levels in the first 180 days of simulation.

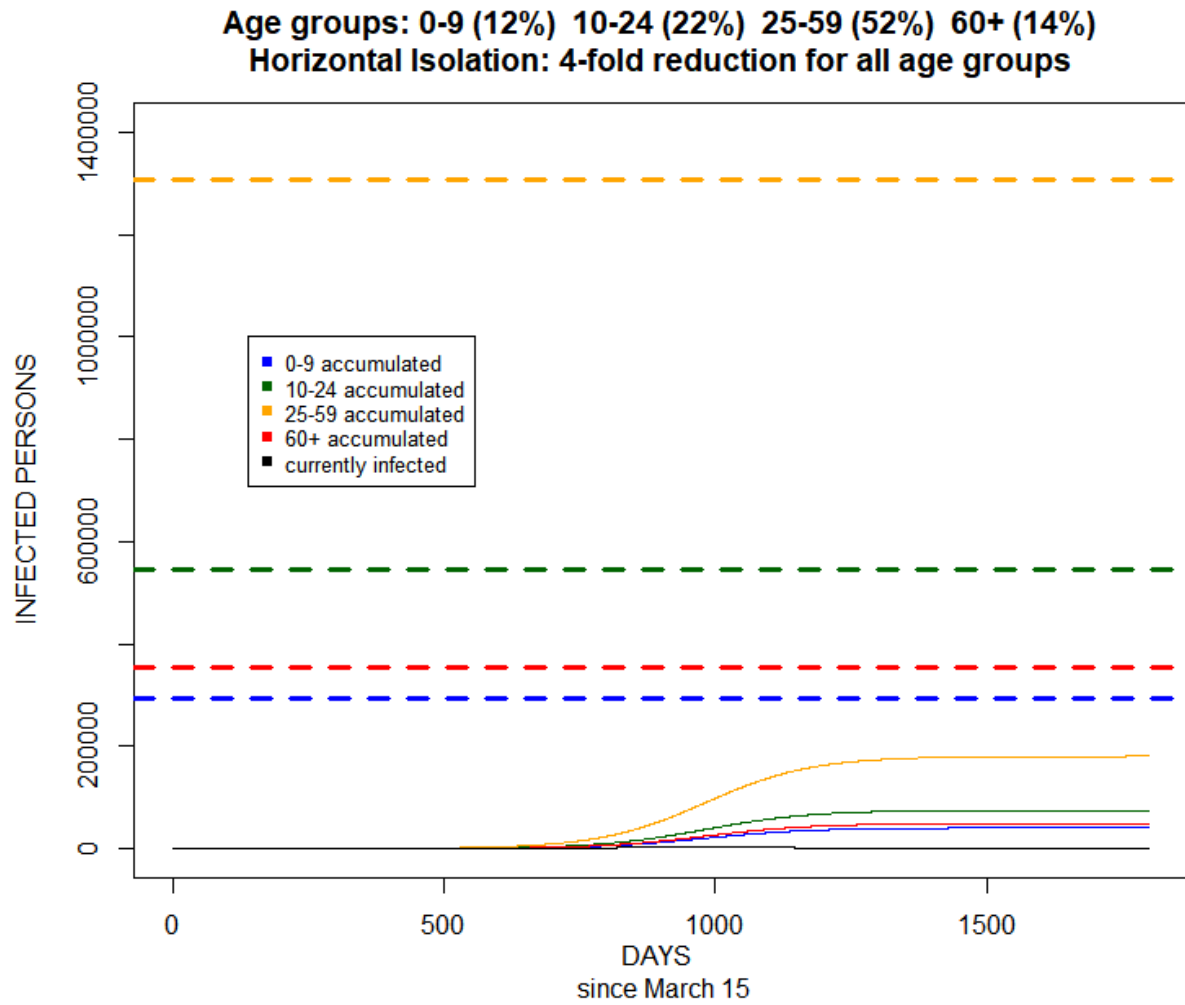


Figure 4: Same as the previous scenario, here displayed for five years, for the horizontal distancing, with a 4-fold social contact intensity factor for all age groups. The epidemic only manifests itself in a reduced way, about 18 months later.

By the end of March, it was estimated that social contacts decreased from 30% to 50% (corresponding to contact intensity between 0.50 and 0.70) in Belo Horizonte ([Google2020]). Using an intensity of $c = 0.55$, corresponding to a value of a $(1/0.55) = 1.8$ -fold reduction in social contact, we obtain the graph of Figure 5. It is immediately apparent that this level of 1.8-fold reduction is not sufficient to deter the epidemic outbreak, as could be observed by the accumulated case's curves reaching more than 85% of the ceiling limit of the number of infected persons for all groups. The peak of simultaneous infections (more than 200 thousand) is reached after about 105 days.

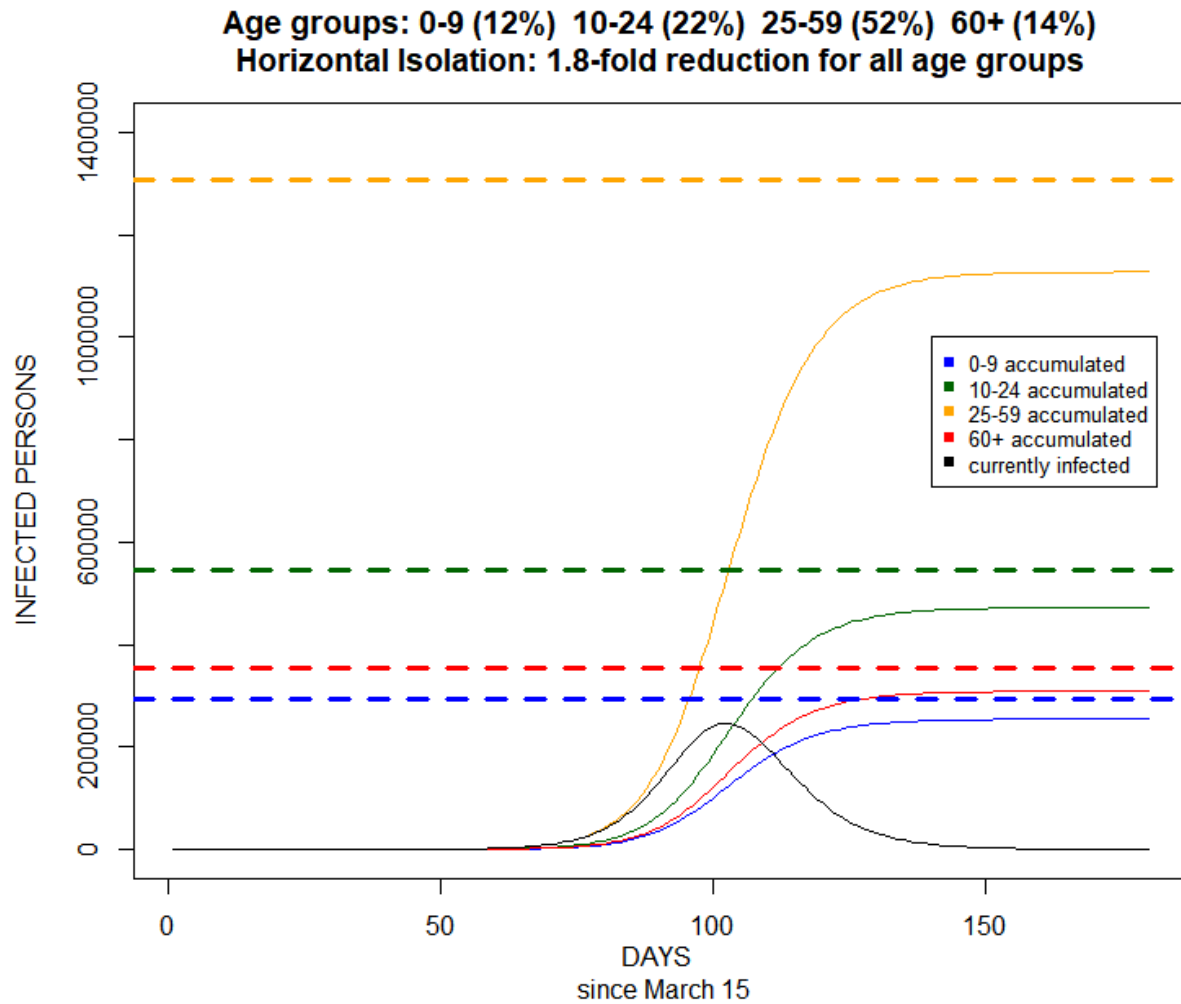


Figure 5: Scenario using a 1.8-fold social contact intensity factor. This is not enough to deter the epidemic.

Estimation of the number of hospitalizations

Using the estimated proportion of COVID-19 infected individuals hospitalized by age group ([Verity, 2020], [Ruan, 2020] and [Garg, 2020]), we have the following rates for the infected population age groups: 0.2% (0-9 years old), 0.4% (10-24 years old), 5% (25-59 years old) and 15% (60+ years old). Figure 6 shows the number of hospital admissions under the five study scenarios of the previous section. Except for the last scenario (4-fold horizontal distancing), the number of required hospital admissions far exceeds the total capacity of the Belo Horizonte city hospitals (about 7,000 beds). Figure 7 displays the accumulated amount of hospital admissions. In the vertical and the 2-fold horizontal scenarios, almost 100 thousand patients would request hospital admission within a short period of three weeks. Note that those simulations do not take into account the population of the larger Belo Horizonte metropolitan area (6 million inhabitants). As almost all hospitals are concentrated within the Belo Horizonte municipality, the predicted inflow of patients from the neighborhood municipalities would exacerbate the problem.

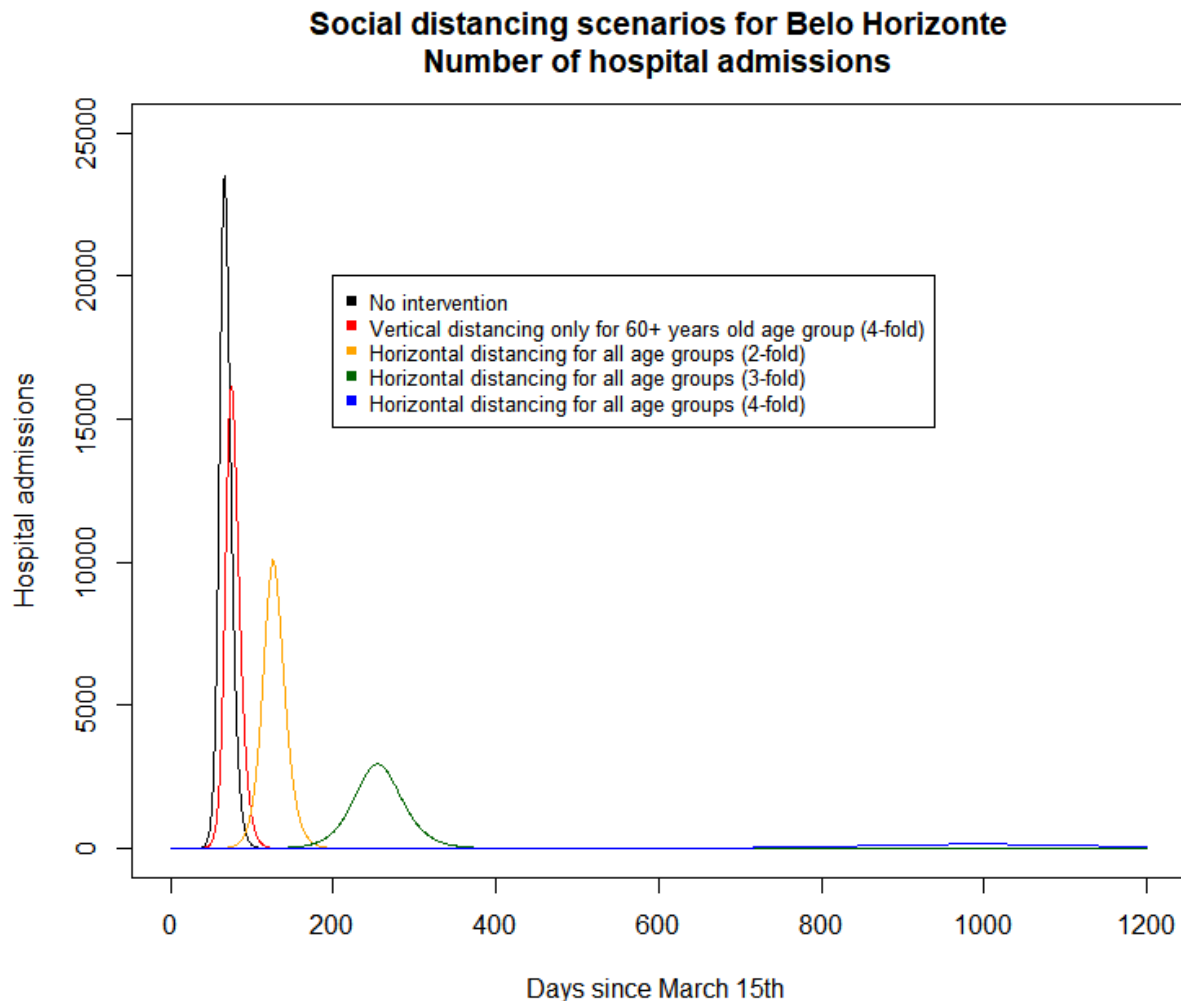


Figure 6: Number of hospital admissions predicted using the five studied scenarios (control, 4-fold vertical, and 2-fold, 3-fold and 4-fold horizontal).

Social distancing scenarios for Belo Horizonte Accumulated number of hospital admissions

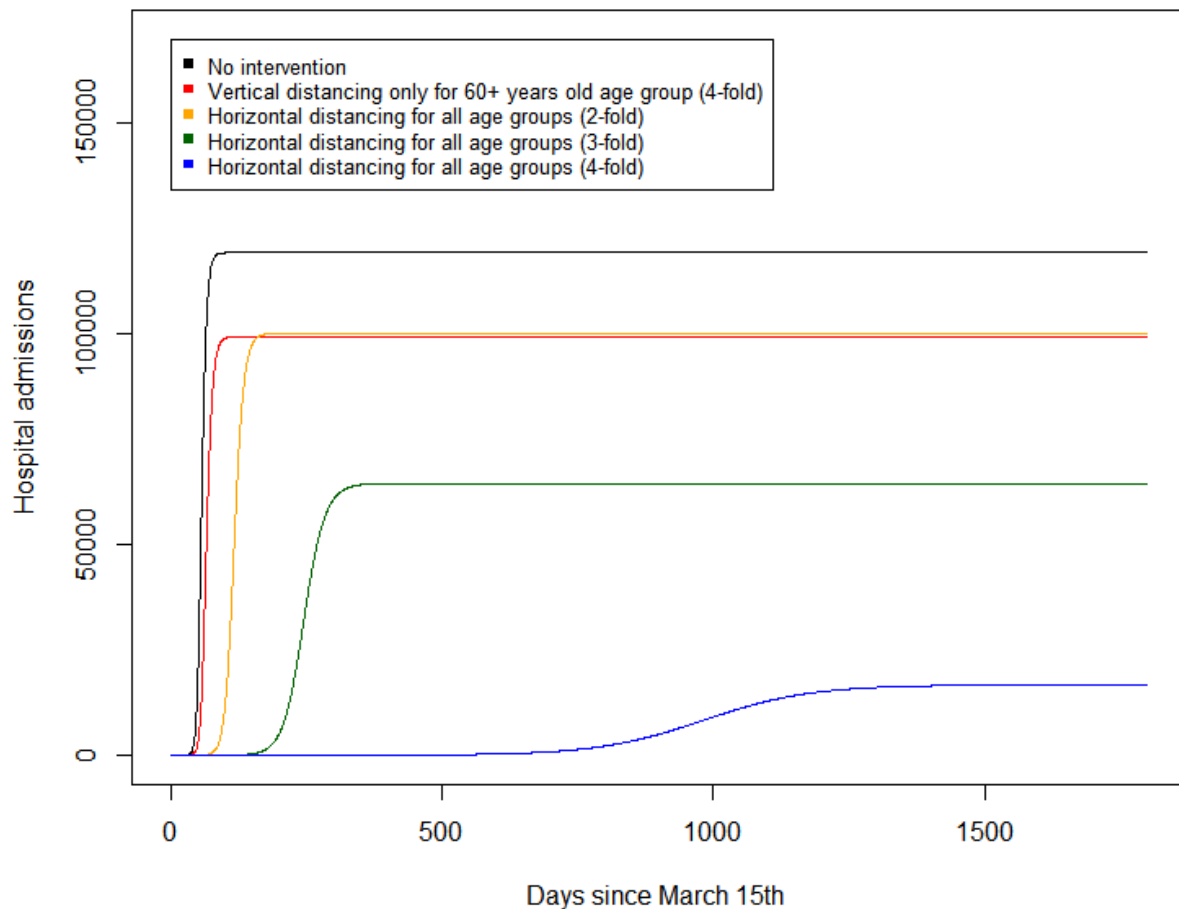


Figure 7: Accumulated number of hospital admissions predicted using the five studied scenarios (control, 4-fold vertical, and 2-fold, 3-fold, and 4-fold horizontal).

Conclusions

The vertical distancing scenario is only marginally better than the situation in which there is no social distancing at all, and much worse than the horizontal distancing scenario, with an equivalent level of reduction in social contact.

Vertical distancing with a 4-fold decrease in social contacts, only for the 60+ years old age group, could not prevent a large number of infected elderly (more than 200 thousand) from appearing, with 350 thousand individuals simultaneously infected. It would also cause a massive flow of patients requiring immediate hospitalization. That would quickly exceed the bed capacity in the Belo Horizonte hospital network.

The proposed horizontal distancing, with a similar 4-fold reduction for all age groups, should slow the surge of cases, postponing the cases' peak for about two years. That should relieve the hospital network, reducing the number of fatal victims, and still allowing future interventions that may occur later (vaccination, new treatments, etc.).

However, this 4-fold reduction is far from being adopted by the general population. Using mobility data for Belo Horizonte, a value of only a 1.8-fold decrease in social contact was achieved during the last two weeks, which is not sufficient to deter the epidemic outbreak. An urgent effort is recommended to improve social contact reduction through a stricter horizontal distancing health policy for several months.

The prediction of requests for hospitalization shows that for the first four studied scenarios (control, 4-fold vertical, and 2-fold and 3-fold horizontal) the inflow of patients coming from Belo Horizonte and neighboring municipalities would create an unprecedented problem, and many people would die from lack of hospital assistance.

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Appendix: The SEIR-Net model

SEIR-Net model description

The SEIR-Net Model is an extension of the SEIR epidemiological model. With the population partitioned into ng groups, the SEIR-Net model consists of a system $4ng$ ordinary differential equations and a set of initial conditions. The system solution consists of $4ng$ time functions in days, which show the evolution of the epidemic's variables (number of susceptible, exposed, infected (reported and unreported) and removed over time).

SEIR-Net Model Parameters

The following variables and parameters are used in the SEIR-Net Model:

N_{total} = Total number of inhabitants ($N = 2500000$ for Belo Horizonte)

ng = number of groups (4 in BH)

$fracg$ = vector of the population fractions of each of the ng groups

((0.1170,0.2176,0.5233,0.1421) in BH)

N = vector of the total number of individuals ($N = N_{total} * fracg$)

S = vector of the total number of susceptible

E = vector of the total number of exposed

I_r = vector of reported number of infected

I_n = vector of the number of unreported infected

I = vector of the total number of infected (reported or not)

R = vector of the total number of removed

(Although S , E , I and R vary, the $N = S + E + I + R$ ratio is always valid.)

μ = reducing factor for the transmission rate of the unreported infected.

(Here $\mu = 1$ was used)

F = Contact Fraction Matrix, where the entry F_{ij} indicates the contact intensity of virus transmission from an individual in the group i to an individual in the group j

B = parameter of virus transmission to individuals in distancing

($B = 1,226$, obtained by adjusting least squares for Belo Horizonte data)

α = Proportion of infected people who will be registered as reported cases

($\alpha = 0.05$: there are 20 times more current cases than reported cases)

Z = Average incubation period ($Z = 3.69$ days, according to [Li2020])

D_r = vector of average duration of the infectious period in reported cases

D_n = vector of Average duration of the infectious period in unreported cases

(D_r and D_n use values of 3.48 days, according to [Li2020])

n_{max} = duration in days of the simulation

SEIR-Net System of Differential Equations

The SEIR-Net model is governed by a system of $4ng$ differential equations. To exemplify, we will explicitly show the equations for four groups in the population ($ng = 4$) with an interaction network between the 4 age groups using the 4×4 matrix $F = [F[i, j]]$, where $F[i, j]$ is the social contact factor that measures contact intensity between an individual in group i who transmits the virus to an individual in group j . The symbol $()'$ indicates derivative in relation to time.

$$I[1] = Ir[1] + \mu * In[1]$$

$$I[2] = Ir[2] + \mu * In[2]$$

$$I[3] = Ir[3] + \mu * In[3]$$

$$I[4] = Ir[4] + \mu * In[4]$$

$$(S[1])' = -(F[1,1]*I[1] + F[2,1]*I[2] + F[3,1]*I[3] + F[4,1]*I[4]) * (B/Ntotal) * (1-I[1]/Ntotal) * S[1]$$

$$(S[2])' = -(F[1,2]*I[1] + F[2,2]*I[2] + F[3,2]*I[3] + F[4,2]*I[4]) * (B/Ntotal) * (1-I[2]/Ntotal) * S[2]$$

$$(S[3])' = -(F[1,3]*I[1] + F[2,3]*I[2] + F[3,3]*I[3] + F[4,3]*I[4]) * (B/Ntotal) * (1-I[3]/Ntotal) * S[3]$$

$$(S[4])' = -(F[1,4]*I[1] + F[2,4]*I[2] + F[3,4]*I[3] + F[4,4]*I[4]) * (B/Ntotal) * (1-I[4]/Ntotal) * S[4]$$

$$(E[1])' = (F[1,1]*I[1] + F[2,1]*I[2] + F[3,1]*I[3] + F[4,1]*I[4]) * (B/Ntotal) * (1-I[1]/Ntotal) * S[1] - E[1]/Z$$

$$(E[2])' = (F[1,2]*I[1] + F[2,2]*I[2] + F[3,2]*I[3] + F[4,2]*I[4]) * (B/Ntotal) * (1-I[2]/Ntotal) * S[2] - E[2]/Z$$

$$(E[3])' = (F[1,3]*I[1] + F[2,3]*I[2] + F[3,3]*I[3] + F[4,3]*I[4]) * (B/Ntotal) * (1-I[3]/Ntotal) * S[3] - E[3]/Z$$

$$(E[4])' = (F[1,4]*I[1] + F[2,4]*I[2] + F[3,4]*I[3] + F[4,4]*I[4]) * (B/Ntotal) * (1-I[4]/Ntotal) * S[4] - E[4]/Z$$

$$(Ir[1])' = \alpha * E[1]/Z - Ir[1]/Dr[1]$$

$$(Ir[2])' = \alpha * E[2]/Z - Ir[2]/Dr[2]$$

$$(Ir[3])' = \alpha * E[3]/Z - Ir[3]/Dr[3]$$

$$(Ir[4])' = \alpha * E[4]/Z - Ir[4]/Dr[4]$$

$$(In[1])' = (1-\alpha) * E[1]/Z - In[1]/Dn[1]$$

$$(In[2])' = (1-\alpha) * E[2]/Z - In[2]/Dn[2]$$

$$(In[3])' = (1-\alpha) * E[3]/Z - In[3]/Dn[3]$$

$$(In[4])' = (1-\alpha) * E[4]/Z - In[4]/Dn[4]$$

Initial conditions

The system described above is initialized with one exposed individual, divided proportionally between the ng groups according to their proportions in the population:

$$E = \text{fracg}$$

$$S = N - \text{fracg}$$

$$I = [0, \dots, 0]$$

$$R = [0, \dots, 0]$$

$$h = 1.0 \text{ (one day step)}$$

$$n_{\max} = 120 \text{ days}$$

Numerical Solution

The SEIR-Net System is solved numerically with the Fourth-order Runge-Kutta method.

Program in R Language

The SEIR-Net method was implemented in R language and can be downloaded from the following link.

<https://drive.google.com/open?id=1Dff5IUcOh3q1uh02izhY5jSK8Ocy5f4P>